Thermo-Electric-Visco-Elastic Material

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ABSTRACT: In this work, we introduce the Seebeck effect in Ohm's law and Thomson heating effect in generalized Fourier's law, to the equations of the linear theory of electro-magneto-thermoviscoelasticity, allowing the second sound effects. A normal mode analysis is used. The resulting formulation is applied to a problem of a rotating thick plate subject to heat on parts of the upper and lower surfaces of the plate that varies exponentially with time. The exact formulas of temperature, displacement, stresses, electric field, magnetic field, and current density are obtained. The considered variables are presented graphically and discussions are made. Seebeck and Peltier effects on thermoelectric viscoelastic material are studied. J Appl Polym Sci 117: 1934–1944, 2010

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INTRODUCTION

The linear theory of elasticity is of paramount importance in the stress analysis of steel, which is the commonest engineering structural material. To a lesser extent, linear elasticity describes the mechanical behavior of the other common solid materials, e.g., concrete, wood, and coal. However, the theory does not apply to the behavior of many of the new synthetic materials of the elastomer and polymer type, e.g., polymethyl-methacrylate (Perspex), polyethylene, and polyvinyl chloride.

With the rapid development of polymer science and plastic industry, as well as the wide use of materials under high temperature in modern technology and application of biology and geology in engineering, the theoretical study and applications in viscoelastic materials has become an important task for solid mechanics.

Linear viscoelastic materials are rheological materials that exhibit time temperature rate-of-loading dependence. When their response is not only a function of the current input but also of the current and past input history, the characterization of the viscoelastic response can be expressed using the convolution (hereditary) integral. A general overview of time-dependent material properties has been presented by Tschoegl.[1] Additionally, a detailed description of the physical response of linear voiscoelastic materials has been explained by Lee and Knauss,[2] based on ramp tests to determine the relaxation modulus, which is a time-domain linear viscoelastic response function. The mechanical-model representation of linear viscoelastic behavior results was investigated by Gross,[3] Staverman and Schwarzl,[4] Alfery and Gurnee[5] and Ferry.[6]

The theory of thermoviscoelasticity and the solutions of some boundary value problems of thermoviscoelasticity were investigated by Ilioushin and Pobedria.[7] The works of Biot,[8,9] Morland and Lee,[10] Tanner,[11] and Huilgol and Phan-Thien[12] made great strides in the last decade in finding solutions for boundary value problems for linear viscoelastic materials including temperature variations in both quasistatic and dynamic problems.

The heat conduction equations for the classical linear uncoupled and coupled thermoelasticity theories are of the diffusion-type predicting infinite speed of propagation for heat wave contrary to physical observations. To eliminate the paradox inherent in the classical theories, the theories of generalized thermoelasticity were developed in attempt to amend the classical thermoelasticity in 1960s. Cattaneo[13] was the first to offer an explicit mathematical correction of the propagation speed defect inherent in Fourier's heat conduction law. Cattaneo's theory allows for the existence of thermal waves, which propagate at finite speeds. The approach used is known as extended irreversible thermodynamics, which introduces time derivative of the heat flux vector, Cauchy stress tensor, and its trace into the classical Fourier law by preserving the entropy principle. Puri and kythe[14] investigated the effects of using the (Maxwell-Cattaneo) model in Stoke's second problem for a viscous fluid. Josef and Preziosi[15,16] give a detail history of heat conduction

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theory. A history of heat conduction also appears in the review article by Dreyer and Struchtrup.[17] They discuss low temperature heat propagation in dielectric solids where second sound effects are present.

Lord and Shulman[18] introduced the theory of generalized thermoelasticity with one relaxation time by postulating a new law of heat conduction to replace the classical Fourier law. This new law contains the flux vector as well as its time derivative. It contains also a new constant that acts as a relaxation time. The heat equation of this theory is of the wave-type, ensuring finite speeds of propagation for heat and elastic waves. The remaining governing equations for this theory, namely, the equations of motions and the constitutive relations, remain the same as those the coupled and the uncoupled theories. Using generalized theory of heat conduction of Lord–Shulman a large number of research workers made valuable contributions in magneto-thermo-elasticity during the last three decades. Öncü and Moodie[19,20] made an analysis of the thermal transient generated by nonuniform sources applied to circular cavities and circular hole in inhomogeneous conductor. Sherief and Ezzat[21] solved a thermal shock half-space problem using asymptotic expansions. Lately, Sherief and Ezzat[22] solved a problem for an infinitely long annular cylinder, while Ezzat[23] solved some problems for perfectly conducting media.

The theory of electro-magneto-thermoviscoelasticity has aroused much interest in many industrial appliances, particularly in nuclear devices, where there exists a primary magnetic field. Various investigations have been carried out by considering the interaction between magnetic, thermal, and strain fields. Analyses of such problems also influence various applications in biomedical engineering as well as in different geomagnetic studies. Misra et al.[24] have studied a one-dimensional uncoupled magnetic-thermoelastic problem in a viscoelastic medium using Maclaurin's approximation method valid for only a specific range of parameters. Ezzat et al.[25,26] introduced the state-space approach for the model of two-dimension equations of generalized thermoviscoelasticity with one and two relaxation times, respectively. A state-space method for the calculation of dynamic response of systems made of viscoelastic materials with exponential-type relaxation kernels was introduced by Menon and Tang.[27] Extension of thermoviscoelastic and magneto-thermo-viscoelastic problems in generalized theory are found to be present in the works of many researchers out of which Mukhopadhyay and Bera,[28] Mukhopadhyay,[29] Karamany and Ezzat,[30] and Rakshit and Mukhopadhyay.[31] The model of the equations of generalized thermoviscoelasticity with the relaxation effects of the volume, with one relaxation time were established by Ezzat.[32] Within the theoretical contributions to the subject are the proofs of uniqueness theorems under different conditions by Ezzat and El Karamany[33,34] and the boundary element formulation was done by El-Karamany and Ezzat.[35] Recently, Ezzat and El-karamany[36] and Ezzat et al.[37] solved some problems in magneto-thermo viscoelasticity of two-temperature.

Quantitative relations for the Seebeck effect were derived from classical mechanics by Drude and later on from quantum physics by Sommerfeld. Theory of the Peltier effect was not developed in such a way, but the relationship between Seebeck coefficient S_α and Peltier coefficient $\prod$ was derived from thermodynamic considerations by Thomson: $\prod = S_\alpha T$.[38]

The Peltier effect is used in thermal analysis and calorimetry for calibration[39,40] and heat flow compensation.[41] Cooling devices on the Peltier effect are used for the design of isothermal microcalorimeters,[42] superconducting magnets,[43] and PC processors.[44] The Peltier effect is included in theoretical and laboratory university courses as one of thermoelectric phenomena.[45]

In this work, we shall formulate the normal mode analysis to a more general model[46] of generalized electromagneto-thermoviscoelastic-coupled two-dimensional problem of a thermally and electrically conducting rotating semispace. The formulas of temperature, displacement, stresses, electric field, magnetic field, and current density are obtained. Application is used to our problem to get the solution in the complete form. The considered variables are presented graphically and comparisons and discussions are made.

BASIC EQUATIONS

Here, we consider a conducting thermoviscoelastic solid of finite conductivity σ_0 permeated by an initial magnetic field H_0. This produces an induced magnetic field h and induced electric field E, which satisfy the linear equations of electromagnetism

$$\varepsilon_{ijk} h_{k,j} = J_i + \varepsilon_o \dot{E}_i, \tag{1}$$

$$\varepsilon_{ijk} E_{k,j} = -\dot{B}_i, \tag{2}$$

$$B_{i,i} = 0,\ D_{i,i} = \rho_e, \tag{3}$$

$$B_i = \mu_0 (H_i + h_i),\ D_i = \varepsilon_0 E_i. \tag{4}$$

The above equations are supplemented by the modified Ohm's law for media with finite conductivity[47]

$$J_i = \sigma_0 \left(E_i + \varepsilon_{ijk} \dot{u}_k H_j \right) - k_o T_i, \tag{5}$$

where k_0 is the coefficient connecting the temperature gradient and the electric current density. The coefficients k_o, π_o, and S_α are interrelated by the relations $k_o = \frac{\kappa}{\pi_o} = \frac{\sigma_o \pi_o}{T_o} = = \sigma_o S_\alpha$ at some reference temperature T_0. These constants will be expressed by different symbols, bearing in mind the above relations.

The governing equations for generalized magneto-thermoviscoelastic in elastic rotation medium, when the effect of Lorentz force is taken into account, consists of:[25]

1. The equations of motion have the form:

$$\sigma_{ji,j} + \mu_o \varepsilon_{ijk} J_k H_j = \rho\left(u_{i,tt} + \Omega_j u_j \Omega_i - \Omega_j^2 u_i + 2\varepsilon_{ijk}\Omega_k u_{j,t}\right)_i, \quad (6)$$

2. The equation of energy in the absence of heat source is given by:

$$\rho T_o \dot{\eta} = -q_{i,i}, \quad (7)$$

The entropy η may be written in terms of temperature, in an isotropic media, as follows

$$\rho\eta = \frac{\rho C_E}{T_o}(T - T_o) + \gamma e, \quad (8)$$

The generalized Fourier's law including the current density effect is given by:[39]

$$q_i + \tau_o \dot{q}_i = -\kappa T_i + \pi_o J_i, \quad (9)$$

where π_0 is the coefficient connecting the current density with the heat flow density.

By eliminating η between (7) and (8), and using (9), we get the equation of heat conduction for the linear theory as follows

$$\kappa\, T_{ii} = \rho C_E\left(\dot{T} + \tau_o \ddot{T}\right) + \gamma\, T_o(\dot{e} + \tau_o \ddot{e}) + \pi_o J_{j,j}, \quad (10)$$

3. The constitutive equation:

$$S_{ij} = \int_0^t R(t-\tau)\frac{\partial e_{ij}(x,\tau)}{\partial \tau} d\tau = \widehat{R}(e_{ij}), \quad (11)$$

with the assumptions

$$\sigma(\widehat{x}, t) = \frac{\partial}{\partial t}\sigma(\widehat{x}, t) = 0, \qquad \varepsilon_{ij}(\widehat{x}, t) = \frac{\partial}{\partial t}\varepsilon_{ij}(\widehat{x}, t) = 0, \quad -\infty < t < 0,$$

$$S_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}, \quad e_{ij} = \varepsilon_{ij} - \frac{e}{3}\delta_{ij}, \quad \sigma_{ji} = \sigma_{ij}, \quad e = \varepsilon_{kk}, \quad (12)$$

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right). \quad (13)$$

and $R(t)$ is the relaxation function, which can be taken in the form:[25]

$$R(t) = 2\mu\left[1 - A\int_0^t e^{-\beta t} t^{\alpha^* - 1} dt\right], \quad R(0) = 2\mu$$

where $0 < a^* <, a > 0, \beta > 0$.

Assuming that the relaxation effects of the volume properties of the material are ignored, we have:

$$\sigma = K[e - 3\alpha_T(T - T_o)]. \quad (14)$$

Substituting eq. (12) into eq. (11) and using eq. (14), we get

$$\sigma_{ij} = \widehat{R}\left(\varepsilon_{ij} - \frac{e}{3}\delta_{ij}\right) + Ke\delta_{ij} - \gamma(T - T_0)\delta_{ij}. \quad (16)$$

From eqs. (6) and (16), it follows that

$$\rho\ddot{u}_i + \mu_o \varepsilon_{rji} J_r H_j = \widehat{R}\left[\frac{1}{2}u_{i,jj} + \frac{1}{6}e_i\right] + Ke_i - \gamma\hat{T}_i, \quad (17)$$

Thus, eqs. (1)–(5), (10), (16), and (17) constitute the field equations and constitutive relations of the linear theory of generalized magneto-viscoelastic thermoelasticity with modified Ohm's law.

Formulation of the problem

We consider a homogenous, isotropic, magneto-viscoelastic generalized thermoelasticity in rotation medium, permeated by an initial magnetic field H_0, acting along the z-axis. The rectangular Cartesian coordinate system(x, y, z) having origin on the surface $x = 0$, with x-axis pointing vertically into the medium is introduced.

For two dimensional problem, we assume the displacement tensor u_i, the initial magnetic field H_i, induced magnetic field $h_{i,}$ the induced electric field E_i, which is normal to the considered magnetic field, and the electric current density J_i is parallel to the electric field as $u_i = (u, v, 0)$, $\Omega = (0, 0, \Omega)$, $H_i = (0, 0, H_0)$, $h_i = (0, 0, h)$, $E_i = (E_1, E_2, 0)$, and $J_i = (J_1, J_2, 0)$. The current density components J_1 and J_2 are given by:

$$J_1 = \sigma_o\left[E_1 + \mu_o H_o \frac{\partial v}{\partial t}\right] - k_o \frac{\partial T}{\partial x}, \quad (18)$$

$$J_2 = \sigma_o\left[E_2 - \mu_o H_o \frac{\partial u}{\partial t}\right] - k_o \frac{\partial T}{\partial y}, \quad (19)$$

We will neglect all second-order quantities, and terms of higher orders eqs. (1)–(17) can be written in the following linearized version, after use the following nondimensional variables (dropping the asterisks for convenience):

$$x^* = c_o\eta_o x, \quad y^* = c_o\eta_o y, \quad u^* = c_o\eta_o u,$$
$$v^* = c_o\eta_o v, \quad t^* = c_o^2\eta_o t,$$
$$\tau_o^* = c_o^2\eta_o\tau, \quad T^* = \frac{\gamma(T-T_o)}{\rho c_o^2}, \quad h^* = \frac{\eta_o h}{\sigma_o\mu_o H_o},$$
$$E_1^* = \frac{\eta_o E_1}{\sigma_o c_o \mu_o^2 H_o}, \quad E_2^* = \frac{\eta_o E_2}{\sigma_o c_o \mu_o^2 H_o},$$
$$J_1^* = \frac{\eta_o J_1}{\sigma_o^2 c_o \mu_o^2 H_o}, \quad J_2^* = \frac{\eta_o J_2}{\sigma_o^2 c_o \mu_o^2 H_o},$$
$$\Omega^* = \frac{\Omega}{\eta_o}, \quad c_o^2 = \frac{\lambda+2\mu}{\rho}, \quad \eta_o = \frac{\rho c_o}{\kappa}.$$

The linear equations of electromagnetism

$$J_1 = E_1 + \frac{1}{v_1}\frac{\partial v}{\partial t} - K_o\frac{\partial T}{\partial x}, \tag{20}$$

$$J_2 = E_2 - \frac{1}{v_1}\frac{\partial u}{\partial t} - K_o\frac{\partial T}{\partial y}, \tag{21}$$

$$\frac{\partial J_1}{\partial x} + \frac{\partial J_2}{\partial y} = -\frac{V^2}{v_1}\left(\frac{\partial E_1}{\partial x} + \frac{\partial E_2}{\partial y}\right), \tag{22}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial J_1}{\partial y} - \frac{\partial J_2}{\partial x}\right) = \frac{1}{v_1}\left(\nabla^2 - V^2\frac{\partial^2}{\partial^2 t}\right)\left(\frac{\partial E_1}{\partial y} - \frac{\partial E_2}{\partial x}\right), \tag{23}$$

$$\frac{\partial E_1}{\partial y} - \frac{\partial E_2}{\partial x} = \frac{\partial h}{\partial t}. \tag{24}$$

The Equations of motions

$$\frac{\partial^2 u}{\partial t^2} = \widehat{R}\left[\frac{1}{2}\nabla^2 u + \frac{1}{6}\frac{\partial e}{\partial x}\right] + \frac{\partial e}{\partial x} - \frac{\partial T}{\partial x} + \Omega^2 u + 2\Omega\frac{\partial v}{\partial t} + v_1^2\varepsilon_2 J_2, \tag{25}$$

$$\frac{\partial^2 v}{\partial t^2} = \widehat{R}\left[\frac{1}{2}\nabla^2 v + \frac{1}{6}\frac{\partial e}{\partial y}\right] + \frac{\partial e}{\partial y} - \frac{\partial T}{\partial y} + \Omega^2 v - 2\Omega\frac{\partial u}{\partial t} - v_1^2\varepsilon_2 J_1, \tag{26}$$

The heat conduction equations

$$\nabla^2 T = \left(\frac{\partial}{\partial t} + \tau_o\frac{\partial^2}{\partial t^2}\right)(T + \varepsilon_1 e) + \pi_1\left(\frac{\partial J_1}{\partial x} + \frac{\partial J_2}{\partial y}\right), \tag{27}$$

The components of the stress tensor

$$\sigma_{xx} = \widehat{R}\left(\frac{\partial u}{\partial x} - \frac{1}{2}\frac{\partial v}{\partial y}\right) + e - T, \tag{28}$$

$$\sigma_{yy} = \widehat{R}\left(\frac{\partial v}{\partial y} - \frac{1}{2}\frac{\partial u}{\partial x}\right) + e - T, \tag{29}$$

$$\sigma_{zz} = -\frac{1}{2}\widehat{R}\,e + e - T, \tag{30}$$

$$\sigma_{xy} = \frac{3}{4}\widehat{R}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tag{31}$$

where

$$v_1 = \frac{\sigma_0\mu_0}{\eta_0}, V^2 = \frac{c_o^2}{c^2}, \varepsilon_1 = \frac{\gamma^2 T_o}{\rho c_o^2\eta_o\kappa}, K_o = \frac{\mu_o H_o k_o}{\gamma\varepsilon_2 v_1^2},$$
$$\varepsilon_2 = \frac{\mu_o H_o^2}{\rho c_o^2}, \pi_1 = \frac{\pi_o\sigma_o\gamma\varepsilon_2 v_1}{K H_o}.$$

Eliminating $E_1$and E_2 between eqs. (21) and (24), we obtain

$$\left(1 + \frac{v_1}{V^2}\right)\left(\frac{\partial J_1}{\partial x} + \frac{\partial J_2}{\partial y}\right) = -v_1\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) - K_o\nabla^2 T, \tag{32}$$

$$\left(\nabla^2 - V^2\frac{\partial^2}{\partial^2 t} - v_1\frac{\partial}{\partial t}\right)\left(\frac{\partial J_1}{\partial y} - \frac{\partial J_2}{\partial x}\right) = \frac{1}{v_1}\left(\nabla^2 - V^2\frac{\partial^2}{\partial^2 t}\right)\frac{\partial e}{\partial t}. \tag{33}$$

Introducing potential functions for each of components of displacement and current density defined by

$$u = \frac{\partial\Phi}{\partial x} + \frac{\partial\Psi}{\partial y}, \quad w = \frac{\partial\Phi}{\partial y} - \frac{\partial\Psi}{\partial x},$$
$$J_1 = \frac{\partial\varsigma}{\partial x} + \frac{\partial\xi}{\partial y}, \quad J_2 = \frac{\partial\varsigma}{\partial y} - \frac{\partial\xi}{\partial x},. \tag{34}$$

Substituting eq. (34) into eqs. (23)–(25), (32), and (33), we obtain

$$\left[\left(\frac{2}{3}\widehat{R} + 1\right)\nabla^2 - \left(\frac{\partial^2}{\partial t^2} - \Omega^2\right)\right]\Phi = T + 2\Omega\frac{\partial\Psi}{\partial t} + v_1^2\varepsilon_2\xi, \tag{35}$$

$$\left[\frac{1}{2}\widehat{R}\,\nabla^2 - \left(\frac{\partial^2}{\partial t^2} - \Omega^2\right)\right]\Psi = -2\Omega\frac{\partial\Phi}{\partial t} - v_1^2\varepsilon_2\varsigma, \tag{36}$$

$$\left[\nabla^2 - \left(\frac{\partial}{\partial t} + \tau_o\frac{\partial^2}{\partial t^2}\right)\right]T = \left(\frac{\partial}{\partial t} + \tau_o\frac{\partial^2}{\partial t^2}\right)\varepsilon_1\nabla^2\Phi + \pi_1\nabla^2\varsigma, \tag{37}$$

$$\left(1 + \frac{v_1}{V^2}\right)\varsigma = -v_1\Psi - K_o T, \tag{38}$$

$$\left(\nabla^2 - V^2\frac{\partial^2}{\partial^2 t} - v_1\frac{\partial}{\partial t}\right)\xi = \frac{1}{v_1}\left(\nabla^2 - V^2\frac{\partial^2}{\partial^2 t}\right)\frac{\partial\Phi}{\partial t}. \tag{39}$$

Equations (36) and (37) can be written as, after elimination ζ by using eq. (38)

$$\left[\frac{1}{2}\widehat{R}\,\nabla^2 - \left(\frac{\partial^2}{\partial t^2} + v_1\varepsilon_3 - \Omega^2\right)\right]\Psi = -2\Omega\frac{\partial\Phi}{\partial t} + \varepsilon_3 K_o T, \tag{40}$$

$$\left[(1 + K_o\pi_2)\nabla^2 - \left(\frac{\partial}{\partial t} + \tau_o\frac{\partial^2}{\partial t^2}\right)\right]T = \left(\frac{\partial}{\partial t} + \tau_o\frac{\partial^2}{\partial t^2}\right)\varepsilon_1\nabla^2\Phi - \pi_2 v_1\nabla^2\Psi, \tag{41}$$

where

$$\varepsilon_3 = \frac{V^2 v_1^2}{V^2 + v_1}\varepsilon_2, \qquad \pi_2 = \frac{V^2 v_1^2}{V^2 + v_1}\pi_1.$$

Normal mode analysis

We consider here that all considered variables can be expressed at the form of plane wave by expressing it in the following exponential form:

$$F(x, y, t) = F^*(x)\exp(\omega t + iqy), \tag{42}$$

where ω is the complex time constant (frequency), q is the wave number in y-axis direction, i is the imaginary unit and F^* is the amplitude of the represented plane waves to the considered variables.

Using eq. (42), we can obtain the following equations from eqs. (35), (37), (39)–(41):

$$\left[\left(\frac{2}{3}\widehat{R}+1\right)\left(D^2 - q^2\right) - \left(\omega^2 - \Omega^2\right)\right]\Phi^* = T^* + 2\omega\Omega\Psi^* + v_1^2\varepsilon_2\xi^*, \tag{43}$$

$$\left[D^2 - \left(V^2\omega^2 + v_1\omega + q^2\right)\right]\xi^* = \frac{\omega}{v_1}\left[D^2 - \left(V^2\omega^2 + q^2\right)\right]\Phi^*. \tag{44}$$

$$\left[\frac{1}{2}\widehat{R}\left(D^2 - q^2\right) - \left(\omega^2 + v_1\varepsilon_3 - \Omega^2\right)\right]\Psi^* = -2\omega\Omega\Phi^* + \varepsilon_3 K_o T^*, \tag{45}$$

$$\left[(1 + K_o\pi_2)\left(D^2 - q^2\right) - \left(\omega + \tau_o\omega^2\right)\right]T^* = \varepsilon_1\left(\omega + \tau_o\omega^2\right)\left(D^2 - q^2\right)\Phi^* - \pi_2 v_1\left(D^2 - q^2\right)\Psi^*, \tag{46}$$

On eqs. (43)–(46) and after some simplification, we obtain

$$\left[D^8 - AD^6 + BD^4 - CD^2 + E\right](\Phi^*, \Psi^*, T^*, \xi^*) = 0, \tag{47}$$

where

$$A = \frac{1}{r\widehat{R}K_1}\left[\widehat{R}(K_o\{n_1 + n_3 r + v_1\varepsilon_2\omega\} + rn_6 - \tau\varepsilon_1) + 2r(K_1 v_1\varepsilon_3\pi_2 + K_1 n_4)\right],$$

$$B = \frac{1}{r\widehat{R}K_1}\left[\begin{array}{l} \mathrm{K}_1(2\varepsilon_3 n_7 + n_8) + n_1 n_6\widehat{R} + \left(n_3\widehat{R}+2n_4\right)(rn_6 - \tau\varepsilon_1) - n_5\widehat{R}\,\varepsilon_1\tau + n_6\widehat{R}\,v_1\omega\varepsilon_2 \\ \quad +4v_1\omega\Omega\pi_2 \end{array}\right],$$

$$C = \frac{1}{r\widehat{R}K_1}\left[\begin{array}{l} 2\mathrm{K}_o\varepsilon_3\left(\begin{array}{l} n_1 v_1\pi_2(\left(n_3 + q^2\right) + n_2 v_1^2\omega\varepsilon_2\pi_2 + n_3\left(q^2 r v_1\pi_2 + 2\omega\Omega\varepsilon_1\tau\right) \\ +\,\omega\left(2n_5\Omega\varepsilon_1\tau + q^2 v_1^2\varepsilon_2\pi_2\right) \end{array}\right) \\ +\,n_2 n_6\widehat{R}\,v_1\omega\varepsilon_2 + 2K_1(n_1 n_3 n_4 + \omega(n_2 n_4 v_1\varepsilon_2 + 4n_3\omega\Omega))) \\ +\,n_1 n_6\left(n_3\widehat{R}+2n_4\right) + n_3\left(2n_4(n_6 r - \varepsilon_1\tau) - n_5\widehat{R}\,\varepsilon_1\tau + 4v_1\omega\Omega\pi_2\right) \\ +\,2(n_4(n_5\varepsilon_1 - n_6 v_1\omega\varepsilon_2)) - 4\Omega\omega\left(2n_6\Omega\omega + q^2 v_1\pi_2\right) \end{array}\right].$$

$$E = \frac{1}{r\widehat{R}K_1}\left[\begin{array}{l} K_o\varepsilon_3\left(n_1 n_3 q^2 v_1\pi_2 + n_2 q^2 v_1^2\omega\varepsilon_2\pi_2 + 2n_3 n_5\omega\Omega\varepsilon_1\tau\right) + n_1 n_3 n_4 n_6 \\ +\,n_2 n_4 n_6 v_1\omega\varepsilon_2 + n_3\left(2\omega\Omega\left(2n_6\omega\Omega + q^2 v_1\pi_2\right) - n_4 n_5\varepsilon_1\tau\right) \end{array}\right].$$

$$n_1 = r\left(q^2\right) + \left(\omega^2 - \Omega^2\right), n_2 = V^2\omega^2 + q^2, n_3 = V^2\omega^2 + v_1\omega + q^2,$$

$$n_4 = \frac{1}{2}\widehat{R}\,q^2 + \left(\omega^2 + v_1\varepsilon_3 - \Omega^2\right),$$

$$n_5 = \tau q^2, n_6 = K_1 q^2 + \tau, \quad \tau = \left(\omega + \tau_o\omega^2\right), K_1 = (1 + K_o\pi_2),\ r = \left(\frac{2}{3}\widehat{R}+1\right),$$

$$n_7 = n_1 v_1\pi 2 + \left(n_3 + q^2\right)(rv_1\pi_2) + v_1^2\omega\varepsilon_2\pi_2 + 2\Omega\omega\varepsilon_1 r,$$

$$n_8 = n_1\left(n_3\widehat{R}+2n_4\right) + n_2\widehat{R}\,v_1\omega\varepsilon_2 + 2\left(n_3 n_4 r + n_4 v_1\omega\varepsilon_2 + 4\omega^2\Omega^2\right).$$

The solution of the eq. (47) satisfying the radiation conditions that Φ^*, ψ^*, T^*, and ξ^* tend to zero as x tends to infinity can be written as

$$(\Phi^*, \Psi^*, T^*, \xi^*)(x, q, \omega) = \sum_{i=1}^{4} (1, \ell_{1i}, \ell_{2i}, \ell_{3i}) A_i e^{-m_i x}, \tag{48}$$

where $A_i(q,\omega)$ are some parameters depending on q and ω,

$$\ell_{1i} = \frac{2\left(K_o\varepsilon_1\varepsilon_3\tau\left(m_i^2 - q^2\right) - 2\omega\Omega\left(K_1 m_i^2 - n_6\right)\right)}{2K_o v_1 \varepsilon_3 \pi_2\left(m_i^2 - q^2\right) + \left(K_1 m_i^2 - n_6\right)\left(\widehat{R}\, m_i^2 - 2n_4\right)},$$

$$\ell_{2i} = \frac{\left(m_i^2 - q^2\right)\left(\varepsilon_1 \widehat{R}\, m_i^2 \tau - 2\left(\varepsilon_1 n_4 \tau - 2v_1\omega\Omega\pi_2\right)\right)}{2K_o v_1 \varepsilon_3 \pi_2\left(m_i^2 - q^2\right) + \left(K_1 m_i^2 - n_6\right)\left(\widehat{R}\, m_i^2 - 2n_4\right)},$$

$$\ell_{3i} = \frac{\omega m_i^2 - n_2}{v_1\left(m_i^2 - n_3\right)}, \qquad i = 1, 2, 3,$$

and m_i (i = 1, 2, 3) are the characteristic roots of the characteristic eq. (47), which is

$$m^8 - Am^6 + Bm^4 - Cm^2 + Em = 0,$$

Now, for obtaining the other potential function of the corresponding current intensity substituting in eq. (38) after using eq. (42)

$$\varsigma^*(x, q, \omega) = \sum_{i=1}^{4} \ell_{4i} A_i e^{-m_i x}, \tag{49}$$

where

$$\ell_{4i} = -\frac{V^2(v_1\ell_{1i} + K_o\ell_{2i})}{V^2 + v_1},$$

By using eq. (34) into (42), we can obtain the displacement components and current density as follows

$$u^*(x, q, \omega) = \sum_{i=1}^{4} (iq\ell_{1i} - m_i) A_i e^{-m_i x}, \tag{50}$$

$$v^*(x, q, \omega) = \sum_{i=1}^{4} (iq + m_i\ell_{1i}) A_i e^{-m_i x}, \tag{51}$$

$$J_1^*(x, q, \omega) = \sum_{i=1}^{4} (iq\ell_{3i} - m_i\ell_{4i}) A_i e^{-m_i x}, \tag{52}$$

$$J_2^*(x, q, \omega) = \sum_{i=1}^{4} (iq\ell_{4i} + m_i\ell_{3i}) A_i e^{-m_i x}. \tag{53}$$

The induced electric field components E_1 and E_2 and induced magnetic field h can be obtained by substituting from (48), (50), and (51) into (26), (27), and (22) respectively, after using eq. (42), we get

$$E_1^*(x, q, \omega) = \frac{1}{v_1}\sum_{i=1}^{4} [iq(v_1\ell_{3i} - \omega) - m_i(v_1\ell_{4i} + v_1 K_o \ell_{2i} + \omega\ell_{1i})] A_i e^{-m_i x}, \tag{54}$$

$$E_2^*(x, q, \omega) = \frac{1}{v_1}\sum_{i=1}^{4} [iq(v_1\ell_{4i} + \omega\ell_{1i} + v_1 K_o \ell_{2i}) + m_i(v_1\ell_{3i} - \omega)] A_i e^{-m_i x}, \tag{55}$$

$$h^*(x, q, \omega) = \frac{1}{v_1}\sum_{i=1}^{4} (\ell_{3i} - \omega)\left(m_i^2 - q^2\right) A_i e^{-m_i x}, \tag{56}$$

Throughout the eqs. (28)–(31), and (50), (51), after using eq. (47), we can obtain the components of stresses:

$$\sigma_{xx} = \frac{1}{2}\sum_{i=1}^{4}\left[q^2\left(\widehat{R} - 2\right) + 2m_i^2\left(1 + \widehat{R}\right) - 3iqm_i \widehat{R}\,\ell_{1i}\right] A_i e^{-m_i x}, \tag{57}$$

$$\sigma_{yy} = \frac{1}{2}\sum_{i=1}^{4}\left[m_i^2\left(2 - \widehat{R}\right) - 2q^2\left(1 + \widehat{R}\right) + 3iqm_i \widehat{R}\,\ell_{1i}\right] A_i e^{-m_i x}, \tag{58}$$

$$\sigma_{zz} = \frac{1}{2}\sum_{i=1}^{4}\left[2\ell_{2i} + \left(2 - \widehat{R}\right)(iq + m_i)\right] A_i e^{-m_i x},$$

$$\sigma_{xy} = -\sum_{i=1}^{4}\left(\ell_{1i}\left(q^2 + m_i^2\right) - 2iqm_i\right) A_i e^{-m_i x}. \tag{59}$$

We shall now use the boundary conditions of the application to evaluate the unknown parameters A_i (i = 1, 2, 3, 4, 5).

APPLICATION

We consider a magneto-viscoelastic thermoelasticity material in elastic rotating medium occupying the semispace region:

$$R = \{(x, y, z) : x \geq 0, -\infty < y, z < \infty\}$$

and the other semispace $R^* = \{(x, y, z): x \leq 0, -\infty < y, z < \infty\}$, is a vacuum, let the surface of R is traction free and subjected to decreasing thermal source with time, which affects on a narrow band of width $2L$ surrounding z-axis and R^* is kept at room temperature T_o and the boundary surface between R and R^* is thermally isolated such that the thermal source affects only on R.

We consider the induced magnetic and electric field intensities in free space. We denote these by h_0, $E_{10,}$ and E_{20}, respectively, hence, Maxwell's equations for region R^* in the nondimensional form, can be simplified to the following:

$$\frac{\partial h_o}{\partial y} = V^2 \frac{\partial E_{1o}}{\partial t}, \tag{60}$$

$$\frac{\partial h_o}{\partial x} = -V^2 \frac{\partial E_{2o}}{\partial t}. \tag{61}$$

$$\frac{\partial E_{1o}}{\partial y} - \frac{\partial E_{2o}}{\partial x} = \frac{\partial h_o}{\partial t}. \tag{62}$$

Once again we apply the normal mode method on these variables to conclude the following equations

$$h_o^*(x) = A_5(q,\omega)e^{nx}, \tag{63}$$

$$E_{1o}^*(x) = \frac{iq}{V^2\omega} A_5(q,\omega)e^{nx}, \tag{64}$$

$$E_{2o}^*(x) = -\frac{n}{V^2\omega} A_5(q,\omega)e^{nx}, \tag{65}$$

where $A_5(q,\omega)$ is a parameter depending on q and ω, and $n = \sqrt{q^2 + V^2\omega^2}$

We apply the following boundary conditions for the purpose of determination of the unknown parameters $A_i(q,\omega)$, ($i = 1, 2, 3, 4, 5$)

1. Thermal boundary condition
   Let $f(y,t)$ be a known function so that
$$\begin{aligned} T(0,y,t) &= f(y,t), \text{ or} \\ T^*(0,q,t) &= f^*(q,t) \end{aligned} \tag{66}$$
2. Mechanical boundary conditions
   Under the assumption that the surface of R is traction free, we can get the following conditions $\sigma_{xx}(0, y, t) = \sigma_{xy}(0, y, t) = 0$.
3. Electromagnetic boundary conditions
   The transverse components of the electric field intensity are continuous across the boundary surface
$$E_2(0,y,t) = E_{20}(0,y,t). \tag{67}$$

The transverse components of the magnetic field intensity are continuous across the boundary surface

$$h(0,y,t) = h_0(0,y,t). \tag{68}$$

With the help of eqs. (48), (56), (57), (59), and (66)–(68), we obtained five equations in four unknown parameters A_i ($i = 1, 2, 3, 4, 5$) in the form:

$$f^*(x,q,\omega) = \sum_{i=1}^{4} \ell_{2i} A_i e^{-m_i x}, \tag{69}$$

$$0 = \sum_{i=1}^{4} (\ell_{3i} - \omega)(m_i^2 - q^2)A_i - v_1 A_5, \tag{70}$$

$$0 = \sum_{i=1}^{4} [iq(v_1\ell_{4i} + \omega\ell_{1i} + v_1 K_o \ell_{2i}) + m_i(v_1\ell_{3i} - \omega)]A_i + \frac{nv_1}{V^2\omega}A_5 \tag{71}$$

$$0 = \sum_{i=1}^{4} \left[q^2\left(\widehat{R} - 2\right) + 2m_i^2(1+\widehat{R}) - 3iqm_i \widehat{R}\, \ell_{1i}\right]A_i, \tag{72}$$

$$0 = \sum_{i=1}^{4} \left(\ell_{1i}(q^2 + m_i^2) - 2iqm_i\right)A_i, \tag{73}$$

Solving eqs. (69)–(73), we obtain the parameters A_i ($i = 1, 2, 3, 4, 5$)

$$A_i = (-1)^{i+1} \frac{f^*(\omega,q)\ell_{2i}^*}{\ell_{21}\ell_{21}^* - \ell_{22}\ell_{22}^* + \ell_{23}\ell_{23}^* - \ell_{24}\ell_{24}^*}, i = 1,2,3,4$$

$$A_5 = \frac{1}{v_1}\sum_{i=1}^{4} (\ell_{3i} - \omega)(m_i^2 - q^2)A_i,$$

where

$$\begin{aligned}
\ell_{5i} &= \frac{1}{v_1}(\ell_{3i} - \omega)(m_i^2 - q^2) \\
&\quad + \frac{V^2\omega}{nv_1}[iq(v_1\ell_{4i} + \omega\ell_{1i} + v_1 K_o\ell_{2i}) + m_i(v_1\ell_{3i} - \omega)] \\
\ell_{6i} &= q^2\left(\widehat{R} - 2\right) + 2m_i^2\left(1 + \widehat{R}\right) - 3iqm_i \widehat{R}\, \ell_{1i} \\
\ell_{7i} &= \ell_{1i}(q^2 + m_i^2) - 2iqm_i \quad i = 1,2,3,4 \\
\ell_{21}^* &= \ell_{52}(\ell_{63}\ell_{74} - \ell_{64}\ell_{73}) + \ell_{53}(\ell_{64}\ell_{72} - \ell_{62}\ell_{74}) \\
&\quad + \ell_{54}(\ell_{62}\ell_{73} - \ell_{63}\ell_{72}), \\
\ell_{22}^* &= \ell_{51}(\ell_{63}\ell_{74} - \ell_{64}\ell_{73}) + \ell_{53}(\ell_{64}\ell_{71} - \ell_{61}\ell_{74}) \\
&\quad + \ell_{54}(\ell_{61}\ell_{73} - \ell_{63}\ell_{71}), \\
\ell_{23}^* &= \ell_{51}(\ell_{62}\ell_{74} - \ell_{64}\ell_{72}) + \ell_{52}(\ell_{64}\ell_{71} - \ell_{61}\ell_{74}) \\
&\quad + \ell_{54}(\ell_{61}\ell_{72} - \ell_{62}\ell_{71}), \\
\ell_{2}^* &= \ell_{51}(\ell_{62}\ell_{74} - \ell_{64}\ell_{72}) + \ell_{52}(\ell_{64}\ell_{71} - \ell_{61}\ell_{74}) \\
&\quad + \ell_{54}(\ell_{61}\ell_{72} - \ell_{62}\ell_{71}),
\end{aligned} \tag{74}$$

By determining these parameters, we have completed solving the problem and now we go to the discussion.

DISCUSSION

The analysis is conducted for a magnesium crystal-like material. Following Ref. [24], the values of physical constants are

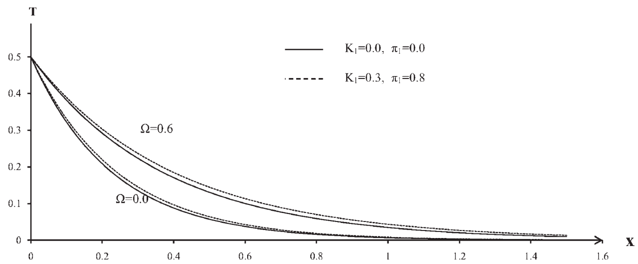

Figure 1 Temperature profiles various values of K_0 and π_1 at $\tau_0 = 0.02$.

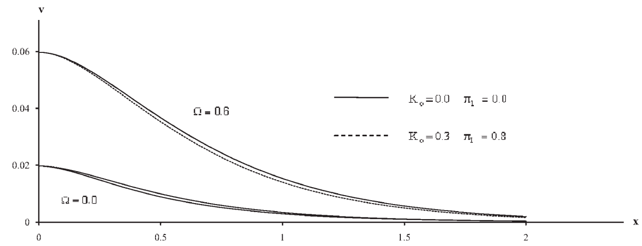

Figure 3 Normal displacement profiles various values of K_0 and π_1 at $\tau_0 = 0.02$.

$\lambda = 9.4 \times 10^{10}\ \mathrm{Nm}^{-2}$,	$\mu = 4.0 \times 10^{10}\ \mathrm{Nm}^{-2}$,	$k = 1.0 \times 10^{10}\ \mathrm{Nm}^{-2}$,
$\rho = 1.74 \times 10^{3}\ \mathrm{gm/cm}^{3}$,	$\gamma = 0.779 \times 10^{-9}\mathrm{N}$,	$j = 0.2 \times 10^{-15}\ \mathrm{cm}^{2}$,
$C_E = 1.04 \times 10^{3}\ \mathrm{kgm}^{-3}$,	$K^{*} = 1.7 \times 10^{2}\ \mathrm{Jm}^{-1}\mathrm{s}^{-1}\mathrm{deg}^{-1}$,	$T_o = 298\ \mathrm{K}$,
$\nu = 3.68 \times 10^{6}\mathrm{Nm}^{-21}\mathrm{deg}^{-1}$,	$\tau_o = 0.02$	$n_o = 0.05$.

we consider the following electric constants for our problem

$\sigma_0 = 9.36 \times 10^5$ Col^2/Cal cm sec, $H_0 = 10^5$ Col/cm sec

$\mu_0 = 4\pi \times 10^{-2}$ dyne sec^2/Col^2, $\varepsilon_0 = 10{-}18/36\pi$ Col^2/dynecm^2.

The function $f(y, t)$ applied on the boundary, is taken as follows $f(y, t) = \theta_0 H(L - |y|)\exp(-bt)$, where θ_0 and b are constants and H is the Heaviside unit step function, putting $f(x, t)$ in normal mode form, we obtain that $f^{*}(q, \omega) = \frac{\theta_o[\cos q\ell - i \sin a\ell]}{\exp[(\omega + b)t]}$, $\quad -L \le \ell \le L$, and t is a certain value of time.

We have that $\omega = \omega_0 + i\omega_0$ then $e^{\omega t} = e^{\omega_o t}(\cos\omega_1 t + i \sin\omega_1 t)$, so for small values of time, we can take ω is real (i.e., $\omega = \omega_0$), in numerical calculations, the other constants of the problem is taken as follows $\omega_0 = 2$, $q = 2$, $\tau_0 = 0.02$, $\theta_0 = 1$, $b = 1$, $k_2 = 0.2$, and $\Omega = 0.6$.

The computations are carried out at time $t = 0.1$, relaxation time $\tau_o = 0.02$, strip width $2L = 0.9 \times 10^3$ on the surface plane $z = 0$. The distribution of non-dimensional variables under two different cases at $\Omega = 0$ and $\Omega = 0.6$ have been shown in Figures 1–9. In these figures the solid line represents magneto-viscoelastic-generalized thermoelastic in a conducting medium with classical Ohm's and Fourier's laws effects ($K_o = 0$, $\pi_1 = 0$), while the dot line represents magneto-viscoelastic-generalized thermoelastic medium with modified Ohm's and Fourier's laws effects ($K_o = 0.3$, $\pi_1 = 0.8$).

The important phenomenon observed in all computations is that the solution of any of the considered functions vanishes identically outside a bounded region of space surrounding the heat source at a distance from it equal to $x^{*}(t)$; say $x^{*}(t)$ is a particular value of x depending only on the choice of t and is the location of the wave front. This demonstrates clearly the difference between the solution corresponding to using Fourier heat equation ($\tau_o = 0.0$) and to using the generalized Fourier case ($\tau_o = 0.02$). In the first and older theory, the waves propagate with infinite speeds, so the value of any of the functions is not identically zero (though it may be very small) for any large value of x. In non-Fourier theory the response to the thermal and mechanical effects does not reach infinity instantaneously but

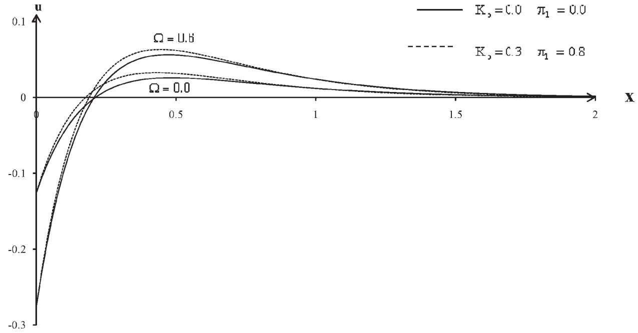

Figure 2 Horizontal displacement profiles various values of K_0 and π_1 at $\tau_0 = 0.02$.

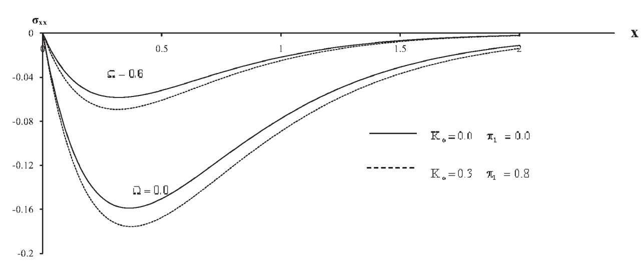

Figure 4 Horizontal stresses profiles various values of K_0 and π_1 at $\tau_0 = 0.02$.

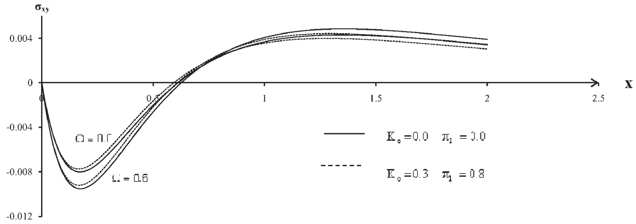

Figure 5 Shearing stresses profiles various values of K_0 and π_1 at $\tau_0 = 0.02$.

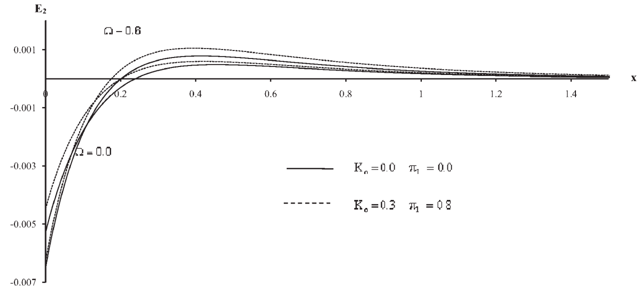

Figure 7 Induced electric field profiles various values of K_0 and π_1 at $\tau_0 = 0.02$.

remains in a bounded region of space given by $0 < x < x^*(t)$ for the semispace problem.

Figure 1 studying the effect of parameters K_0, π_1 on the temperature. We noticed from this figure that the four curves start from the origin point then decreases till it tends to zero at $x > 1.5$ and the values of T in modified Ohm's and Fourier's laws are higher than these in the classical case.

The components of displacement u and v are illustrated graphically in Figures 2 and 3. It is noticed that the curve of horizontal displacement largest in modified Ohm's and Fourier's laws than these in the classical case, while the normal displacement in the event of extended and decreases in the case with modified Ohm's and Fourier's laws.

It can be found from Figures 2 and 4 that rotation acts to significantly decrease the magnitude of the real part of displacement and stress.

In Figures 4 and 5 it is noticed that the absolute values of normal stresses σ_{xx} are increased in the modified model as compared with the values for classical case, while shearing stresses σ_{xy} is decreases.

Figures 6 and 7 describe the variations of the induced magnetic and electric fields, respectively, it is evident that the values of both fields are increased in the modified model.

The Seebeck and Peltier effects are shown to be closely related within the new thermodynamic model applied recently to the quantitative theory of the Seebeck coefficient. In this work, the model was developed for the evaluation of the Seebeck and Peltier coefficients. The gradual decrease of thermopower with temperature as shown in Figure 8 has also been reported by Huston,[48] Ambia et al.[49] and Patankar et al.[50] In Figure 9, we observe that the Peltier coefficient is proportional to the temperature at constant value of Seebeck coefficient. These results agrees with the expectation by the first Thomson relation $\prod = S_\alpha T$.[51]

CONCLUSIONS

The trend of variations of the temperature distribution T, transverse displacement u, normal displacement v, normal stress σ_{xx} and shearing stresses σ_{xy} are quite different on the application of new model and old model. The medium, which is taken, is affected by parameters K_0, π_1, and magnetic field, more on the application of modified Ohm's and Fourier's laws in comparison to the application of classical model. The increasing in the values of temperature may be explained as the lost heat generating from the movement of electric current, this heat

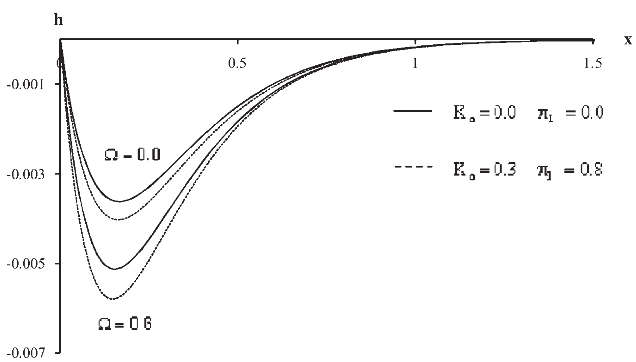

Figure 6 Induced magnetic field profiles various values of K_0 and π_1 at $\tau_0 = 0.02$.

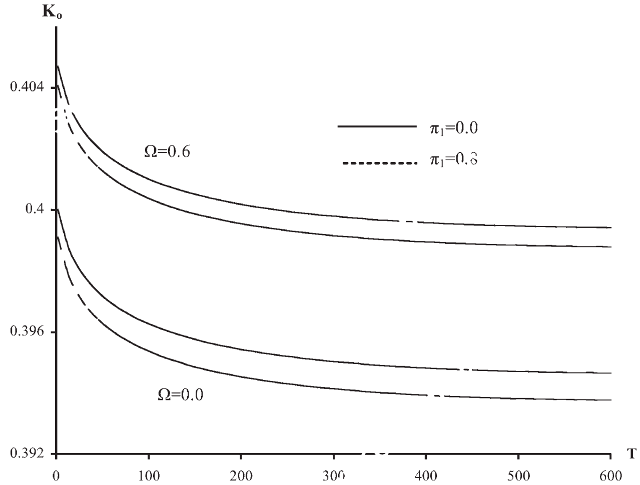

Figure 8 Variation of Seebeck coefficient with temperature.

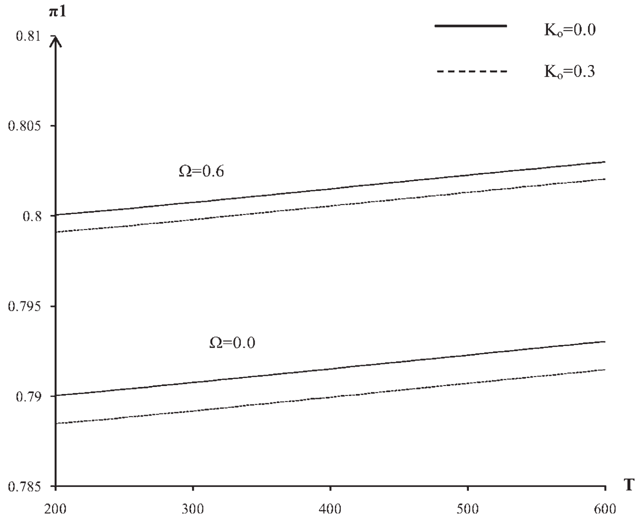

Figure 9 Variation of Peltier coefficient with temperature.

may be the main reason to make that the deformation of the medium tends to be normal, whereas the components of electric and magnetic fields record values greater than the values recorded in the classical model.

NOMENCLATURE

D_i	electric displacement tensor
E_i	induced electric field tensor
J_i	current density tensor
H_i	magnetic intensity tensor
H_i	induced magnetic field tensor
H_0	initial magnetic field vector
ε_0	dielectric constant
μ_0	magnetic permeability
ρ_e	charge density
σ_0	electric conductivity
ε_{ijk}	permutation symbol
λ,μ	Lame's constants
ρ	Density
C_E	specific heat at constant strain
t	time
T	absolute temperature
T_0	reference temperature chosen so that $\lvert T - T_0 \rvert \ll 1$
ε_{ij}	components of strain tensor
S_{ij}	components of stress deviator tensor
e_{ij}	components of strain deviator tensor
u_i	components of displacement tensor
Ωi	angular velocity tensor
R	relaxation function
k	Thermal conductivity
A,β,a^*	empirical constants
K	$\lambda + \frac{2}{3}\mu$ bulk modulus
τ_0	relaxation time
α_T	coefficient of linear thermal expansion
Q	the strength of the applied heat source per unit mass
γ	$3K\,\alpha_T$
c^2	$\frac{1}{\varepsilon_o\mu_o}$ light speed
δ_0	nondimensional constant for adjustment the reference temperature
c_o^2	K/ρ
η_0	$(\rho c_E)/K$
ε	$(\delta_0\gamma)/(\rho c_E)$
S_α	Seebeck coefficient
π_o	Peltier coefficient
k_0	$\frac{\kappa}{\pi_o}$

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